# New High-Efficiency Source of a Three-Photon $W$ State and its Full Characterization Using Quantum State Tomography

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We propose and demonstrate a new scheme of high-efficiency generation of the three-photon polarization-entangled $W$ state, which is a typical three-qubit entangled state. The high efficiency has enabled the first full characterization of the state by quantum state tomography. We have analyzed the obtained state and observed its nature of tripartite entanglement and robustness of entanglement.

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Entanglement has a key role in not only fundamental quantum physics but also quantum information processing. Bipartite entanglement has been well studied both theoretically and experimentally. Multipartite entanglement is a more recent topic of importance in the field of quantum information. In the case of tripartite system, genuine tripartite entangled states are classified into two classes by the equivalence under stochastic local operations and classical communications [1]. Typical states of these classes are the Greenberger-Horne-Zeilinger (GHZ) state $|\mathrm{GHZ}\rangle = 1/\sqrt{2}(|000\rangle + |111\rangle)$ and the $W$ state $|W\rangle = 1/\sqrt{3}(|001\rangle + |010\rangle + |100\rangle)$. These tripartite entangled states have been realized experimentally using several resources [2,3]. However, especially in the case of multiphoton experiments, all the performed measurements have been indirect; i.e., they are able to obtain only partial information about the generated multiphoton states [4]. Quantum state tomography [7] is a method to reconstruct a density matrix of an experimentally obtained state and thus is equal to performing full characterization. Full characterization enables any kind of criterion to be applied to the state, and thus is very important for both fundamental study and application. An obstacle to full characterization of a multiphoton state is its low generation efficiency. As an example, the previous work of generating a three-photon $W$ state [3] suffers too low a generation efficiency to perform full characterization. Moreover, because a photon is the primary candidate for a resource of quantum communication, a high-efficiency source of multiphoton entanglement is in itself highly desired in this field. In this Letter, we report on a scheme of the generation of a three-photon $W$ state with much higher efficiency than the previous work and full characterization of the experimentally obtained state.

A schematic diagram of the experimental setup is shown in Fig. 1. We utilized the process of the optical parametric amplification (OPA) in a nonlinear crystal [8,9]. The interaction Hamiltonian in a nonlinear crystal in the presence of a strong pump pulse is given as $H = \kappa(a_H^\dagger b_V^\dagger + a_V^\dagger b_H^\dagger) + \mathrm{H.c.}$, where $\kappa$ is the coupling constant and $a^\dagger$ ($b^\dagger$) is a creation operator for photons in a spatial mode $a$ ($b$). The subscript $H$ ($V$) represents horizontal (vertical) polarization, and H.c. is the hermitian conjugate. Now coherent light is incident on the crystal, with sufficiently weak power so that it can be approximated as a single photon. The polarization of the coherent light is assumed to be horizontal. Accordingly, the first-order term of the state after the OPA process becomes

$$-i\kappa t(a_H^\dagger b_V^\dagger + a_V^\dagger b_H^\dagger)|0\rangle_a|H\rangle_b \tag{1}$$

$$= -i\kappa t(|H\rangle_a|HV\rangle_b + \sqrt{2}|V\rangle_a|HH\rangle_b), \tag{2}$$

where $H$ ($V$) in kets denotes a photon with horizontal (vertical) polarization and 0 denotes vacuum. We assumed that the seed light is incident on mode $b$. The factor $\sqrt{2}$ is due to the stimulated emission. Photons in mode $b$ are injected to a nonpolarizing 50/50 beam splitter so that they are split into modes $b$ and $c$. Photons in mode $a$ transmit a set of Brewster windows whose transmission coefficients for horizontal light ($t_H$) and vertical light ($t_V$) are adjusted to have the relation $t_H = 2t_V$. After all the processes, we select the case where a detector set on each output mode detects one photon. Finally the total state becomes a three-

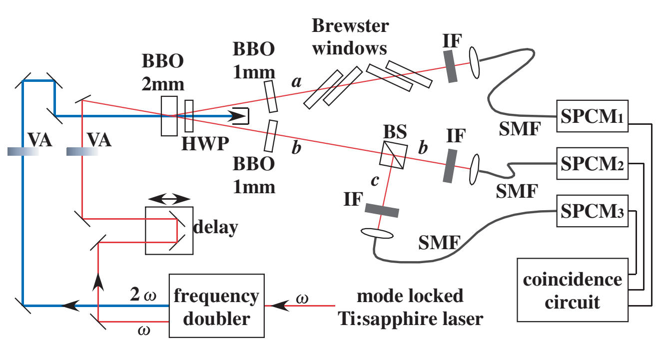


FIG. 1 (color online). Schematic diagram of experimental setup; VA, variable attenuator; HWP, half-wave plate; BS, nonpolarizing 50/50 beam splitter; IF, 3 nm-bandwidth interference filter; SMF, single mode fiber; SPCM$_i$ ($i = 1, 2, 3$), single photon counting module.

 

photon $W$ state

$$1\sqrt{3}(|HHV\rangle_{abc} + |HVH\rangle_{abc} + |VHH\rangle_{abc}) \tag{3}$$

where qubits are encoded as $0 \rightarrow H$ and $1 \rightarrow V$ and the state is normalized.

In our experiment, UV pulses with a central wavelength at 390 nm from a frequency-doubled mode-locked Ti:sapphire laser were incident on a 2 mm thick $\beta$-barium borate (BBO) crystal as the pump source of the OPA, where polarization-entangled photon pairs are created into modes $a$ and $b$ in the absence of the seed light [10]. Residual fundamental pulses with a central wavelength at 780 nm from the frequency doubler were used as the seed of the OPA. To adjust the relative timing of the pump and seed pulses, a delay line was inserted in the path of the seed beam. A half-wave plate and two auxiliary 1 mm thick BBO crystals were used to compensate the spacial and temporal walk-off of down-converted photons. Photons in each output mode were filtered by a 3 nm-bandwidth interference filter, coupled into a single mode fiber, and detected by single photon counting modules (Perkin Elmer SPCM-AQR-14). The power of the pump and the seed light should be carefully adjusted to remove some unwanted multiphoton terms, which induce noise [11]. The values of mean photon number of the seed light and the generation efficiency of a down-converted photon pair per pulse were set for $\alpha \sim 0.1$ and $\beta \sim 0.0025$, respectively. The average count rate of threefold coincidence was 1.45 counts per second for the pump power set at 80 mW. These values are about 40 times higher and 10 times lower than those of the previous report, i.e., 125 counts per hour and 700 mW, respectively [3]. Note that the limitation of the generation efficiency of the present setup is mainly caused by the photon-statistical nature of coherent state, while the previous work suffered the limitation of low nonlinearity of the crystal. Thus when the weak coherent light is replaced by an output of a single photon source, which is now progressively developed [12], the generation efficiency will further dramatically increase. This implies that the present scheme supplies not only practical but also potential improvement of efficiency.

To verify the indistinguishability of the seed and down-converted photons, stimulated emission within the nonlinear crystal was observed [8,9]. A polarizer was inserted in each output port and set so that it makes projection onto $V$, $H$, and $H$ for modes $a$, $b$, and $c$, respectively, and threefold coincidence events of detectors were recorded with the delay line scanned. The experimental data are shown in Fig. 2. In the ideal case, the ratio between the maximum value of the curve (perfect indistinguishability) and the base value (perfect distinguishability) should become 2 because of the factor $\sqrt{2}$ in Eq. (2). The observed ratio is $1.82 \pm 0.05$, which is comparable with the ideal one. This result implies that the spatial overlap of the seed and down-converted light is good enough. All the following experiments were performed at the zero-delay point.

For full characterization of the obtained state, quantum state tomography was performed [7]. For three-qubit tomography, $2^3 \times 2^3 = 64$ projective measurements are required. Each output mode is projected on the basis $|H\rangle, |V\rangle, |P\rangle = 1/\sqrt{2}(|H\rangle + |V\rangle)$, and $|R\rangle = 1/\sqrt{2}(|H\rangle - i|V\rangle)$ in each measurement, which was performed by inserting a quarter-wave plate and a polarizer in each output mode and setting their axes properly. Threefold coincidence counts were measured and the density matrix was reconstructed from 64 sets of count numbers. Each data point was measured for 550 sec. Note that the pump power and the coupling efficiency of the measurement setup should be stable through a train of 64 projective measurements, because fluctuation of detection efficiency induces errors which cannot be modeled by density matrix. We confirmed that such a condition was satisfied during all the measurement. The reconstructed density matrix is shown in Fig. 3. It can be obviously seen that, within the diagonal parts, $|HHV\rangle_{abc}\langle HHV|$, $|HVH\rangle_{abc}\langle HVH|$, and $|VHH\rangle_{abc}\langle VHH|$ are dominant. Also within the off-diagonal parts, cross terms of the dominant diagonal parts

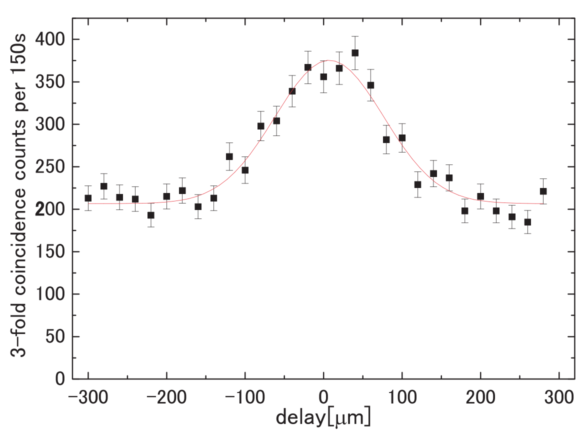


FIG. 2 (color online). Observed data of threefold coincidence counts of photons. Vertically, horizontally, and horizontally polarized photons are detected in modes $a$, $b$, and $c$, respectively.

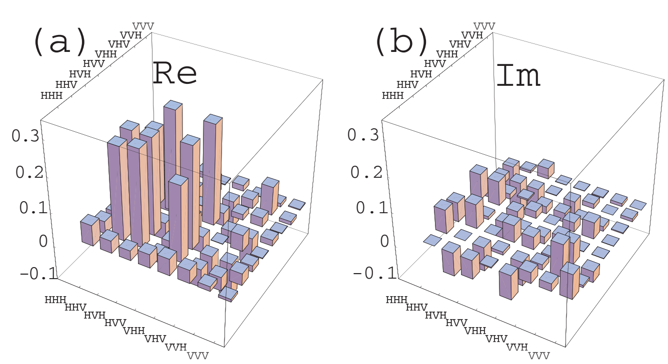


FIG. 3 (color online). Experimentally determined density matrix of photons in modes $a$, $b$, and $c$. (a) The real part of the density matrix. (b) The imaginary part of the density matrix.

are dominant. These facts imply that the three terms $|HHV\rangle_{abc}$, $|HVH\rangle_{abc}$, and $|VHH\rangle_{abc}$ are well superposed and other terms are almost negligible. The calculated values of fidelity of the density matrix with the genuine $W$ state $\langle W|\rho_{\rm exp}|W\rangle$ and purity $\mathrm{Tr}[\rho_{\rm exp}^2]$ were $0.80 \pm 0.04$ and $0.88 \pm 0.10$, respectively. From the parameters $\alpha$ and $\beta$, we considered possible spurious threefold coincidence counts caused by multiphoton process which mainly consists of (i) two photons from both the weak coherent light and the down-converted light and (ii) four photons from the down-converted light and zero photon from the weak coherent light, and the calculated maximum values of fidelity and purity are 0.93 and 0.87, respectively. Here we emphasize that the successful tomographic reconstruction is exclusively due to the high efficiency of the present setup.

We observed genuine tripartite entanglement by witness operators [13] $\mathcal{W}_W^{(1)} = 2/3\mathbb{I} - |W\rangle\langle W|$ and $\mathcal{W}_W^{(2)} = 1/2\mathbb{I} - |\overline{\mathrm{GHZ}}\rangle\langle\overline{\mathrm{GHZ}}|$, where $\mathbb{I}$ is the identity operator of three-qubit system and $|\overline{\mathrm{GHZ}}\rangle = 1/\sqrt{2}(|\bar{0}\,\bar{0}\,\bar{0}\rangle + |\bar{1}\,\bar{1}\,\bar{1}\rangle)$ with $|\bar{0}\rangle = 1/\sqrt{2}(|0\rangle + i|1\rangle)$ and $|\bar{1}\rangle = 1/\sqrt{2}(-|0\rangle + i|1\rangle)$. Negativity of the expectation value of each of these operators is a *sufficient* condition for being tripartite entanglement [14]. The obtained expectation values of the witness operators are $\mathrm{Tr}(\mathcal{W}_W^{(1)}\rho_{\rm exp}) = -0.138 \pm 0.043$ and $\mathrm{Tr}(\mathcal{W}_W^{(2)}\rho_{\rm exp}) = -0.070 \pm 0.036$, which are explicitly negative. From this result, we can conclude that a genuine tripartite entangled state has been observed.

We also tested robustness of entanglement, which is a notable feature of the three-qubit $W$ state [1]. The reduced density matrix of modes $b$ and $c$ ($\mathrm{Tr}_a\rho_{\rm exp}$) is shown in Fig. 4. The Peres-Horodecki criterion was applied to this density matrix [15]. The minimum eigenvalue of the partial transpose of the density matrix is $-0.17 \pm 0.03$, which is comparable with the theoretical value of $-0.20$. According to the criterion, this negative value definitively shows that the reduced bipartite system is entangled. We have also performed the same procedure to the reduced density matrix of modes $a$, $b$ ($\mathrm{Tr}_c\rho_{\rm exp}$) and modes $a$, $c$ ($\mathrm{Tr}_b\rho_{\rm exp}$), and obtained eigenvalues $-0.15 \pm 0.03$ and $-0.19 \pm 0.03$, respectively, which is also unambiguous proof of entanglement. The GHZ state does not have such a feature; i.e., its subsystems are not entangled.

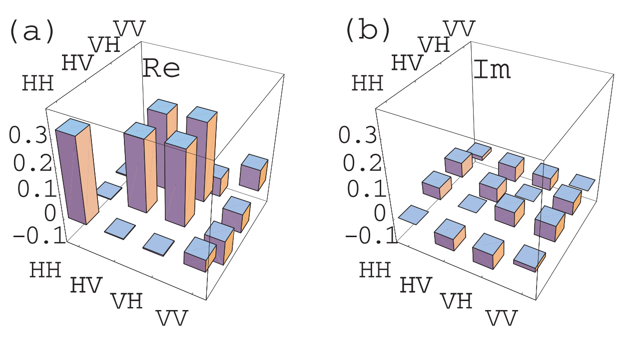


FIG. 4 (color online). Reduced density matrix of photons in modes $b$ and $c$, where mode $a$ is traced out. (a) The real part of the density matrix. (b) The imaginary part of the density matrix.

The present scheme involves post-selection technique, where in the absence of quantum nondemolition photon-number measurement, the desired state can be obtained only after measurement, i.e., being destroyed. This condition limits real application to quantum information processing. However, there are various useful schemes of measurement-based quantum information processing such as quantum cryptography [16], linear optics quantum computation [17], and one-way quantum computation [6,18], which does not suffer such a condition. Thus the present scheme can be considered to have sufficient availability for really useful quantum information processing.

We briefly mention the connection between the present scheme of generating a three-photon $W$ state and the previous work of quantum cloning [8], which has a setup similar to the present one. In the present setup, the stimulated emission is a result of coherence between the seed and the down-converted light, and enhances probability of existence of input polarization (currently $H$). This effect is the essence of optimal quantum cloning. However, it also enhances asymmetry among the three output port, though the $W$ state has perfect symmetry. Therefore the Brewster windows are inserted to symmetrize the output state. Moreover, the present scheme still works as a kind of quantum cloning machine, i.e., quantum phase-covariant cloning machine [19,20] for input (seed) states $\cos\theta|H\rangle + e^{i\phi}\sin\theta|V\rangle$ with fixed $\theta$, though the achievable fidelity is not optimal [21]. In contrast, the output states obtained in the previous work of quantum cloning [8] also belong to the $W$ class and thus are tripartite entangled states [22]. The present setup has also similarity to that of several protocols such as, quantum universal-NOT [9,23], quantum teleportation [24]. Thus the present work is expected to accelerate further study of relationship among these processes, multipartite entanglement, and also the process of the optical parametric amplification.

The present setup can further be naturally extended to several possible applications. First, it can be used to generate states similar to the $W$ state. Especially, the state which can be used as a source of quantum telecloning [25], $1/\sqrt{6}(2|HHV\rangle_{abc} - |HVH\rangle_{abc} - |VHH\rangle_{abc})$, can be immediately obtained by removing the Brewster windows and changing relative phases of the terms in Eq. (3), which is easily realized by tilting a compensation crystal by a small angle. Second, it can naturally be extended to that of generation of $n$-photon $W$ state ($n \geq 4$) [26]. Especially, the case of $n = 4$ is readily feasible. This is achieved by replacing weak coherent light by two-photon state from type-I down-conversion. Third, by analogy with the process of quantum cloning or quantum universal-NOT [24], the setup can be reconstructed as a teleportation-like scheme, which is implemented using a beam splitter instead of the OPA. This fact implies that the concept of the present scheme can be applied to fermionic resources such as electrons. Fourth, a three-photon $W$ state can in itself be utilized for quantum communication protocols such as quantum secret sharing [16] and quantum teleportation [27].

In conclusion, we have experimentally demonstrated a new scheme for generating the three-photon polarization-entangled $W$ state with so high efficiency that it enabled the first full characterization of the state by quantum state tomography. We exhibited the first observation of the nature of robustness of entanglement of the $W$ state [28]. We showed that the present scheme has sufficient availability, though it involves post-selection technique. We also showed the connection between the present scheme and that of the previous work of quantum cloning [8]. The present work is the first demonstration of tomographic characterization of multiphoton entanglement except for only the very recent works [5,6] and will provide an important step toward fundamental understanding of multipartite entanglement and its application to quantum information processing.

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